

2026  
CFA<sup>®</sup>  
Exam Prep

# SchweserNotes<sup>™</sup>

Fixed Income, Derivatives, and  
Alternative Investments

Level II Book 4

KAPLAN SCHWESER

# Book 4: Fixed Income, Derivatives, and Alternative Investments

SchweserNotes™ 2026

Level II CFA®

**KAPLAN**  **SCHWESER**

SCHWESERNOTES™ 2026 LEVEL II CFA® BOOK 4: FIXED INCOME, DERIVATIVES, AND ALTERNATIVE INVESTMENTS

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# CONTENTS

---

Learning Outcome Statements (LOS)

## **FIXED INCOME**

---

### **READING 23**

#### **The Term Structure and Interest Rate Dynamics**

Exam Focus

Introduction

Module 23.1: Spot and Forward Rates, Part 1

Module 23.2: Spot and Forward Rates, Part 2

Module 23.4: Spread Measures

Module 23.5: Term Structure Theory

Module 23.6: Yield Curve Risks and Economic Factors

Key Concepts

Answer Key for Module Quizzes

### **READING 24**

#### **The Arbitrage-Free Valuation Framework**

Exam Focus

Module 24.1: Binomial Trees, Part 1

Module 24.2: Binomial Trees, Part 2

Module 24.3: Interest Rate Models

Key Concepts

Answer Key for Module Quizzes

### **READING 25**

#### **Valuation and Analysis of Bonds With Embedded Options**

Exam Focus

Module 25.1: Types of Embedded Options

Module 25.2: Valuing Bonds With Embedded Options, Part 1

Module 25.3: Valuing Bonds With Embedded Options, Part 2

Module 25.4: Option-Adjusted Spread

Module 25.5: Duration

Module 25.6: Key Rate Duration

Module 25.7: Capped and Floored Floaters

Module 25.8: Convertible Bonds

Key Concepts  
Answer Key for Module Quizzes

## **READING 26**

### **Credit Analysis Models**

Exam Focus

Module 26.1: Credit Risk Measures

Module 26.2: Analysis of Credit Risk

Module 26.3: Credit Scores and Credit Ratings

Module 26.4: Structural and Reduced Form Models

Module 26.5: Credit Spread Analysis

Module 26.6: Credit Spread

Module 26.7: Credit Analysis of Securitized Debt

Key Concepts

Answer Key for Module Quizzes

## **READING 27**

### **Credit Default Swaps**

Exam Focus

Module 27.1: CDS Features and Terms

Module 27.2: Factors Affecting CDS Pricing

Module 27.3: CDS Usage

Key Concepts

Answer Key for Module Quizzes

Topic Quiz: Fixed Income

## **DERIVATIVES**

---

## **READING 28**

### **Pricing and Valuation of Forward Commitments**

Exam Focus

Module 28.1: Pricing and Valuation Concepts

Module 28.2: Pricing and Valuation of Equity Forwards

Module 28.3: Pricing and Valuation of Fixed Income Forwards

Module 28.4: Pricing and Valuation of Forward Rate Agreements

Module 28.5: Pricing and Valuation of Interest Rate Swaps

Module 28.6: Currency Swaps

Module 28.7: Equity Swaps

Key Concepts

Answer Key for Module Quizzes

## **READING 29**

### **Valuation of Contingent Claims**

Exam Focus

Module 29.1: The Binomial Model

Module 29.2: Two Period Binomial Model and Put-Call Parity

Module 29.3: American Options

Module 29.4: Hedge Ratio

Module 29.5: Interest Rate Options

Module 29.6: Black-Scholes-Merton and Swaptions

Module 29.7: Option Greeks and Dynamic Hedging

Key Concepts

Answer Key for Module Quizzes

Topic Quiz: Derivatives

## **ALTERNATIVE INVESTMENTS**

---

### **READING 30**

#### **Introduction to Commodities and Commodity Derivatives**

Exam Focus

Module 30.1: Introduction and Theories of Return

Module 30.2: Analyzing Returns and Index Construction

Key Concepts

Answer Key for Module Quizzes

### **READING 31**

#### **Overview of Types of Real Estate Investment**

Exam Focus

Module 31.1: Real Estate Features

Module 31.2: Value Drivers and Property Types

Module 31.3: Due Diligence, Valuation, and Indexes

Key Concepts

Answer Key for Module Quizzes

### **READING 32**

#### **Investments in Real Estate Through Publicly Traded Securities**

Exam Focus

Module 32.1: Investments in Real Estate Through Publicly Traded Securities

Key Concepts

Answer Key for Module Quizzes

### **READING 33**

#### **Hedge Fund Strategies**

Exam Focus

Module 33.1: Overview of Hedge Fund Strategies

Module 33.2: Equity, Event-Driven, and Relative Value Strategies

Module 33.3: Opportunistic, Specialist, and Multi-Manager Strategies

Module 33.4: Factor Models and Portfolio Impact of Hedge Funds

Key Concepts

Answer Key for Module Quizzes

Topic Quiz: Alternative Investments

Formulas

Index

# Learning Outcome Statements (LOS)

## 23. The Term Structure and Interest Rate Dynamics

The candidate should be able to:

- a. describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve. (page 2)
- b. describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping. (page 5)
- c. describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management. (page 7)
- d. describe the strategy of rolling down the yield curve. (page 10)
- e. explain the swap rate curve and why and how market participants use it in valuation. (page 11)
- f. calculate and interpret the swap spread for a given maturity. (page 13)
- g. describe short-term interest rate spreads used to gauge economy-wide credit risk and liquidity risk. (page 14)
- h. explain traditional theories of the term structure of interest rates and describe the implications of each theory for forward rates and the shape of the yield curve. (page 17)
- i. explain how a bond's exposure to each of the factors driving the yield curve can be measured and how these exposures can be used to manage yield curve risks. (page 20)
- j. explain the maturity structure of yield volatilities and their effect on price volatility. (page 22)
- k. explain how key economic factors are used to establish a view on benchmark rates, spreads, and yield curve changes. (page 22)

## 24. The Arbitrage-Free Valuation Framework

The candidate should be able to:

- a. explain what is meant by arbitrage-free valuation of a fixed-income instrument. (page 31)
- b. calculate the arbitrage-free value of an option-free, fixed-rate coupon bond. (page 32)
- c. describe a binomial interest rate tree framework. (page 33)
- d. describe the process of calibrating a binomial interest rate tree to match a specific term structure. (page 38)
- e. describe the backward induction valuation methodology and calculate the value of a fixed-income instrument given its cash flow at each node. (page 34)
- f. compare pricing using the zero-coupon yield curve with pricing using an arbitrage-free binomial lattice. (page 39)
- g. describe pathwise valuation in a binomial interest rate framework and calculate the value of a fixed-income instrument given its cash flows along each path. (page 41)
- h. describe a Monte Carlo forward-rate simulation and its application. (page 42)
- i. describe term structure models and how they are used. (page 44)

## 25. Valuation and Analysis of Bonds With Embedded Options

The candidate should be able to:

- a. describe fixed-income securities with embedded options. (page 55)
- b. explain the relationships between the values of a callable or puttable bond, the underlying option-free (straight) bond, and the embedded option. (page 56)
- c. describe how the arbitrage-free framework can be used to value a bond with embedded options. (page 57)
- d. explain how interest rate volatility affects the value of a callable or puttable bond. (page 62)
- e. explain how changes in the level and shape of the yield curve affect the value of a callable or puttable bond. (page 62)
- f. calculate the value of a callable or puttable bond from an interest rate tree. (page 57)
- g. explain the calculation and use of option-adjusted spreads. (page 63)
- h. explain how interest rate volatility affects option-adjusted spreads. (page 65)
- i. calculate and interpret effective duration of a callable or puttable bond. (page 66)
- j. compare effective durations of callable, puttable, and straight bonds. (page 67)
- k. describe the use of one-sided durations and key rate durations to evaluate the interest rate sensitivity of bonds with embedded options. (page 68)

- l. compare effective convexities of callable, putable, and straight bonds. (page 70)
- m. calculate the value of a capped or floored floating-rate bond. (page 71)
- n. describe defining features of a convertible bond. (page 75)
- o. calculate and interpret the components of a convertible bond's value. (page 76)
- p. describe how a convertible bond is valued in an arbitrage-free framework. (page 78)
- q. compare the risk–return characteristics of a convertible bond with the risk–return characteristics of a straight bond and of the underlying common stock. (page 79)

## **26. Credit Analysis Models**

The candidate should be able to:

- a. explain expected exposure, the loss given default, the probability of default, and the credit valuation adjustment. (page 91)
- b. explain credit scores and credit ratings. (page 98)
- c. calculate the expected return on a bond given transition in its credit rating. (page 99)
- d. explain structural and reduced-form models of corporate credit risk, including assumptions, strengths, and weaknesses. (page 100)
- e. calculate the value of a bond and its credit spread, given assumptions about the credit risk parameters. (page 103)
- f. interpret changes in a credit spread. (page 106)
- g. explain the determinants of the term structure of credit spreads and interpret a term structure of credit spreads. (page 108)
- h. compare the credit analysis required for securitized debt to the credit analysis of corporate debt. (page 111)

## **27. Credit Default Swaps**

The candidate should be able to:

- a. describe credit default swaps (CDS), single-name and index CDS, and the parameters that define a given CDS product. (page 120)
- b. describe credit events and settlement protocols with respect to CDS. (page 122)
- c. Explain the principles underlying and factors that influence the market's pricing of CDS. (page 123)
- d. describe the use of CDS to manage credit exposures and to express views regarding changes in the shape and/or level of the credit curve. (page 125)
- e. describe the use of CDS to take advantage of valuation disparities among separate markets, such as bonds, loans, equities, and equity-linked instruments. (page 126)

## **28. Pricing and Valuation of Forward Commitments**

The candidate should be able to:

- a. describe how equity forwards and futures are priced, and calculate and interpret their no-arbitrage value. (page 138)
- b. describe the carry arbitrage model without underlying cashflows and with underlying cashflows. (page 134)
- c. describe how interest rate forwards and futures are priced, and calculate and interpret their no-arbitrage value. (page 144)
- d. describe how fixed-income forwards and futures are priced, and calculate and interpret their no-arbitrage value. (page 141)
- e. describe how interest rate swaps are priced, and calculate and interpret their no-arbitrage value. (page 151)
- f. describe how currency swaps are priced, and calculate and interpret their no-arbitrage value. (page 155)
- g. describe how equity swaps are priced, and calculate and interpret their no-arbitrage value. (page 158)

## **29. Valuation of Contingent Claims**

The candidate should be able to:

- a. describe and interpret the binomial option valuation model and its component terms. (page 169)
- b. describe how the value of a European option can be analyzed as the present value of the option's expected payoff at expiration. (page 169)
- c. identify an arbitrage opportunity involving options and describe the related arbitrage. (page 177)
- d. calculate the no-arbitrage values of European and American options using a two-period binomial model. (page 169)

- e. calculate and interpret the value of an interest rate option using a two-period binomial model. (page 180)
- f. identify assumptions of the Black–Scholes–Merton option valuation model. (page 183)
- g. interpret the components of the Black–Scholes–Merton model as applied to call options in terms of a leveraged position in the underlying. (page 183)
- h. describe how the Black–Scholes–Merton model is used to value European options on equities and currencies. (page 185)
- i. describe how the Black model is used to value European options on futures. (page 186)
- j. describe how the Black model is used to value European interest rate options and European swaptions. (page 187)
- k. interpret each of the option Greeks. (page 189)
- l. describe how a delta hedge is executed. (page 195)
- m. describe the role of gamma risk in options trading. (page 196)
- n. define implied volatility and explain how it is used in options trading. (page 196)

### **30. Introduction to Commodities and Commodity Derivatives**

The candidate should be able to:

- a. compare characteristics of commodity sectors. (page 207)
- b. compare the life cycle of commodity sectors from production through trading or consumption. (page 210)
- c. contrast the valuation of commodities with the valuation of equities and bonds. (page 211)
- d. describe types of participants in commodity futures markets. (page 211)
- e. analyze the relationship between spot prices and futures prices in markets in contango and markets in backwardation. (page 212)
- f. compare theories of commodity futures returns. (page 213)
- g. describe, calculate, and interpret the components of total return for a fully collateralized commodity futures contract. (page 215)
- h. contrast roll return in markets in contango and markets in backwardation. (page 216)
- i. describe how commodity swaps are used to obtain or modify exposure to commodities. (page 216)
- j. describe how the construction of commodity indexes affects index returns. (page 218)

### **31. Overview of Types of Real Estate Investment**

The candidate should be able to:

- a. compare important real estate investment features for valuation purposes. (page 225)
- b. explain economic value drivers of real estate investments and their role in a portfolio. (page 231)
- c. discuss the distinctive investment characteristics of commercial property types. (page 232)
- d. explain the due diligence process and valuation approaches for real estate investments. (page 234)
- e. discuss real estate investment indexes, including their construction and potential biases. (page 238)

### **32. Investments in Real Estate Through Publicly Traded Securities**

The candidate should be able to:

- a. discuss types of publicly traded real estate securities. (page 247)
- b. justify the use of net asset value per share (NAVPS) in valuation of publicly traded real estate securities and estimate NAVPS based on forecasted cash net operating income. (page 250)
- c. describe the use of funds from operations (FFO) and adjusted funds from operations (AFFO) in REIT valuation. (page 252)
- d. calculate and interpret the value of a REIT share using the net asset value, relative value (price-to-FFO and price-to-AFFO), and discounted cash flow approaches. (page 254)
- e. explain advantages and disadvantages of investing in real estate through publicly traded securities compared to private vehicles. (page 257)

### **33. Hedge Fund Strategies**

The candidate should be able to:

- a. discuss how hedge fund strategies may be classified. (page 268)
- b. discuss investment characteristics, strategy implementation, and role in a portfolio of equity-related hedge fund strategies. (page 269)
- c. discuss investment characteristics, strategy implementation, and role in a portfolio of event-driven hedge fund strategies. (page 272)
- d. discuss investment characteristics, strategy implementation, and role in a portfolio of relative value hedge fund strategies. (page 275)

- e. discuss investment characteristics, strategy implementation, and role in a portfolio of opportunistic hedge fund strategies. (page 278)
- f. discuss investment characteristics, strategy implementation, and role in a portfolio of specialist hedge fund strategies. (page 282)
- g. discuss investment characteristics, strategy implementation, and role in a portfolio of multi-manager hedge fund strategies. (page 285)
- h. describe how factor models may be used to understand hedge fund risk exposures. (page 290)
- i. evaluate the impact of an allocation to a hedge fund strategy in a traditional investment portfolio. (page 291)

## READING 23

# THE TERM STRUCTURE AND INTEREST RATE DYNAMICS

### EXAM FOCUS

This topic review discusses the theories and implications of the term structure of interest rates. In addition to understanding the relationships between spot rates, forward rates, yield to maturity, and the shape of the yield curve, be sure you become familiar with concepts like the Z-spread, the TED spread and the MRR-OIS spread. Interpreting the shape of the yield curve in the context of the theories of the term structure of interest rates is always important for the exam. Also pay close attention to the concept of key rate duration.

### INTRODUCTION

The financial markets both impact and are controlled by interest rates. Understanding the term structure of interest rates (i.e., the graph of interest rates at different maturities) is one key to understanding the performance of an economy. In this reading, we explain how and why the term structure changes over time.

**Spot rates** are the annualized market interest rates for a single payment to be received in the future. Generally, we use spot rates for government securities (risk-free) to generate the spot rate curve. Spot rates can be interpreted as the yields on zero-coupon bonds, and for this reason we sometimes refer to spot rates as *zero-coupon rates*. A **forward rate** is an interest rate (agreed to today) for a loan to be made at some future date.



#### PROFESSOR'S NOTE

While most of the LOS in this topic review have *describe* or *explain* as the command words, we will still delve into numerous calculations, as it is difficult to really understand some of these concepts without getting in to the mathematics behind them.

## MODULE 23.1: SPOT AND FORWARD RATES, PART 1

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**LOS 23.a: Describe relationships among spot rates, forward rates, yield to maturity, expected and realized returns on bonds, and the shape of the yield curve.**

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### Spot Rates

The price today of \$1 par, zero-coupon bond is known as the discount factor, which we will call  $P_T$ . Because it is a zero-coupon bond, the spot interest rate is the yield to maturity of this payment, which we represent as  $S_T$ . The relationship between the discount factor  $P_T$  and the spot rate  $S_T$  for maturity  $T$  can be expressed as:

$$P_T = \frac{1}{(1 + S_T)^T}$$

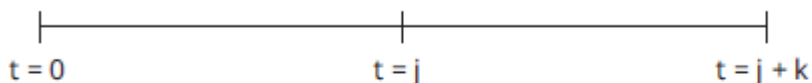
The term structure of spot rates—the graph of the spot rate  $S_T$  versus the maturity  $T$ —is known as the **spot yield curve** or **spot curve**. The shape and level of the spot curve changes continuously with the market prices of bonds.

### Forward Rates

The annualized interest rate on a loan to be initiated at a future period is called the **forward rate** for that period. The term structure of forward rates is called the **forward curve**. (Note that forward curves and spot curves are mathematically related—we can derive one from the other.)

We will use the following notation:

$f(j,k)$  = the annualized interest rate applicable on a  $k$ -year loan starting in  $j$  years.



$F_{(j,k)}$  = the forward price of a \$1 par zero-coupon bond maturing at time  $j+k$  delivered at time  $j$ .

$$F_{(j,k)} = \frac{1}{[1 + f(j,k)]^k}$$

### Yield to Maturity

As we've discussed, the **yield to maturity (YTM)** or yield of a zero-coupon bond with maturity  $T$  is the spot interest rate for a maturity of  $T$ . However, for a coupon bond, if the spot rate curve is not flat, the YTM will not be the same as the spot rate.

#### EXAMPLE: Spot rates and yield for a coupon bond

Compute the price and yield to maturity of a three-year, 4% annual-pay, \$1,000 face value bond given the following spot rate curve:  $S_1 = 5\%$ ,  $S_2 = 6\%$ , and  $S_3 = 7\%$ .

### Answer:

1. Calculate the price of the bond using the spot rate curve:

$$\text{Price} = \frac{40}{(1.05)} + \frac{40}{(1.06)^2} + \frac{1040}{(1.07)^3} = \$922.64$$

2. Calculate the yield to maturity ( $y_3$ ):

$$N = 3; PV = -922.64; PMT = 40; FV = 1,000; \text{CPT I/Y} \rightarrow 6.94$$

$$y_3 = 6.94\%$$

Note that the yield on a three-year bond is a weighted average of three spot rates, so in this case we would expect  $S_1 < y_3 < S_3$ . The yield to maturity  $y_3$  is closest to  $S_3$  because the par value dominates the value of the bond and therefore  $S_3$  has the highest weight.

## Expected and Realized Returns on Bonds

Expected return is the ex-ante holding period return that a bond investor expects to earn.

The expected return will be equal to the bond's yield only when *all three* of the following are true:

- The bond is held to maturity.
- All payments (coupon and principal) are made on time and in full.
- All coupons are reinvested at the original YTM.

The second requirement implies that the bond is option-free and there is no default risk.

The last requirement, reinvesting coupons at the YTM, is the least realistic assumption. If the yield curve is not flat, the coupon payments will not be reinvested at the YTM and the expected return will differ from the yield.

Realized return on a bond refers to the actual return that the investor experiences over the investment's holding period. Realized return is based on actual reinvestment rates.

## The Forward Pricing Model

The **forward pricing model** values forward contracts based on arbitrage-free pricing.

Consider two investors.

Investor A purchases a \$1 face value, zero-coupon bond maturing in  $j+k$  years at a price of  $P_{(j+k)}$ .

Investor B enters into a  $j$ -year forward contract to purchase a \$1 face value, zero-coupon bond maturing in  $k$  years at a price of  $F_{(j,k)}$ . Investor B's cost today is the present value of the cost:  $PV[F_{(j,k)}]$  or  $P_j F_{(j,k)}$ .

Because the \$1 cash flows at  $j+k$  are the same, these two investments should have the same price, which leads to the forward pricing model:

$$P_{(j+k)} = P_j F_{(j,k)}$$

Therefore:

$$F_{(j,k)} = \frac{P_{(j+k)}}{P_j}$$

### EXAMPLE: Forward pricing

Calculate the forward price two years from now for a \$1 par, zero-coupon, three-year bond given the following spot rates.

The two-year spot rate,  $S_2 = 4\%$ .

The five-year spot rate,  $S_5 = 6\%$ .

#### Answer:

Calculate discount factors  $P_j$  and  $P_{(j+k)}$ .

$$P_j = P_2 = 1 / (1 + 0.04)^2 = 0.9246$$

$$P_{(j+k)} = P_5 = 1 / (1 + 0.06)^5 = 0.7473$$

The forward price of a three-year bond in two years is represented as  $F_{(2,3)}$

$$F_{(j,k)} = P_{(j+k)} / P_j$$

$$F_{(2,3)} = 0.7473 / 0.9246 = 0.8082$$

In other words, \$0.8082 is the price agreed to today, to pay in two years, for a three-year bond that will pay \$1 at maturity.



#### PROFESSOR'S NOTE

In the Derivatives portion of the curriculum, the forward price is computed as the future value (for  $j$  periods) of  $P_{(j+k)}$ . It gives the same result and can be verified using the data in the previous example by computing the future value of  $P_5$  (i.e., compounding for two periods at  $S_2$ ).  $FV = 0.7473(1.04)^2 = \$0.8082$ .

## The Forward Rate Model

The **forward rate model** relates forward and spot rates as follows:

$$[1 + S_{(j+k)}]^{(j+k)} = (1 + S_j)^j [1 + f(j,k)]^k$$

or

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

This model is useful because it illustrates how forward rates and spot rates are interrelated.

This equation suggests that the forward rate  $f(2,3)$  should make investors indifferent between buying a five-year zero-coupon bond versus buying a two-year zero-coupon bond and at maturity reinvesting the principal for three additional years.

**EXAMPLE: Forward rates**

Suppose that the two-year and five-year spot rates are  $S_2 = 4\%$  and  $S_5 = 6\%$ .

Calculate the implied three-year forward rate for a loan starting two years from now [i.e.,  $f(2,3)$ ].

**Answer:**

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

$$[1 + f(2,3)]^3 = [1 + 0.06]^5 / [1 + 0.04]^2$$

$$f(2,3) = 7.35\%$$

Note that the forward rate  $f(2,3) > S_5$  because the yield curve is upward sloping.

If the yield curve is upward sloping, [i.e.,  $S_{(j+k)} > S_j$ ], then the forward rate corresponding to the period from  $j$  to  $k$  [i.e.,  $f(j,k)$ ] will be greater than the spot rate for maturity  $j+k$  [i.e.,  $S_{(j+k)}$ ]. The opposite is true if the curve is downward sloping.

**LOS 23.b: Describe how zero-coupon rates (spot rates) may be obtained from the par curve by bootstrapping.**

A **par rate** is the yield to maturity of a bond trading at par. Par rates for bonds with different maturities make up the **par rate curve** or simply the **par curve**. By definition, the par rate will be equal to the coupon rate on the bond. Generally, par curve refers to the par rates for government or benchmark bonds.

By using a process called **bootstrapping**, spot rates or zero-coupon rates can be derived from the par curve. Bootstrapping involves using the output of one step as an input to the next step. We first recognize that (for annual-pay bonds) the one-year spot rate ( $S_1$ ) is the same as the one-year par rate. We can then compute  $S_2$  using  $S_1$  as one of the inputs. Continuing the process, we can compute the three-year spot rate  $S_3$  using  $S_1$  and  $S_2$  computed earlier. Let's clarify this with an example.

**EXAMPLE: Bootstrapping spot rates**

Given the following (annual-pay) par curve, compute the corresponding spot rate curve:

Maturity	Par Rate
1	1.00%
2	1.25%
3	1.50%

**Answer:**

$S_1 = 1.00\%$  (given directly).

If we discount each cash flow of the bond using its yield, we get the market price of the bond. Here, the market price is the par value. Consider the 2-year bond.

$$100 = \frac{1.25}{(1.0125)} + \frac{101.25}{(1.0125)^2}$$

Alternatively, we can also value the 2-year bond using spot rates:

$$100 = \frac{1.25}{(1 + S_1)} + \frac{101.25}{(1 + S_2)^2} = \frac{1.25}{(1.01)} + \frac{101.25}{(1 + S_2)^2}$$

$$100 = 1.2376 + \frac{101.25}{(1 + S_2)^2}$$

$$98.7624 = \frac{101.25}{(1 + S_2)^2}$$

Multiplying both sides by  $[(1 + S_2)^2 / 98.7624]$ , we get  $(1 + S_2)^2 = 1.0252$ .

Taking square roots, we get  $(1 + S_2) = 1.01252$ .  $S_2 = 0.01252$  or  $1.252\%$

Similarly,

$$100 = \frac{1.50}{(1 + S_1)} + \frac{1.50}{(1 + S_2)^2} + \frac{101.50}{(1 + S_3)^3}$$

Using the values of  $S_1$  and  $S_2$  computed earlier,

$$100 = \frac{1.50}{(1.01)} + \frac{1.50}{(1.01252)^2} + \frac{101.50}{(1 + S_3)^3}$$

$$100 = 2.9483 + \frac{101.50}{(1 + S_3)^3}$$

$$97.0517 = \frac{101.50}{(1 + S_3)^3}$$

$$(1 + S_3)^3 = 1.0458$$

$$(1 + S_3) = 1.0151 \text{ and hence } S_3 = 1.51\%$$



**MODULE QUIZ 23.1**

1. When the yield curve is downward sloping, the forward curves are *most likely* to lie:
  - A. above the spot curve.
  - B. below the spot curve.
  - C. either above or below the spot curve.
2. The model that equates buying a long-maturity zero-coupon bond to entering into a forward contract to buy a zero-coupon bond that matures at the same time is known as the:
  - A. forward rate model.
  - B. forward pricing model.

C. forward arbitrage model.

## MODULE 23.2: SPOT AND FORWARD RATES, PART 2

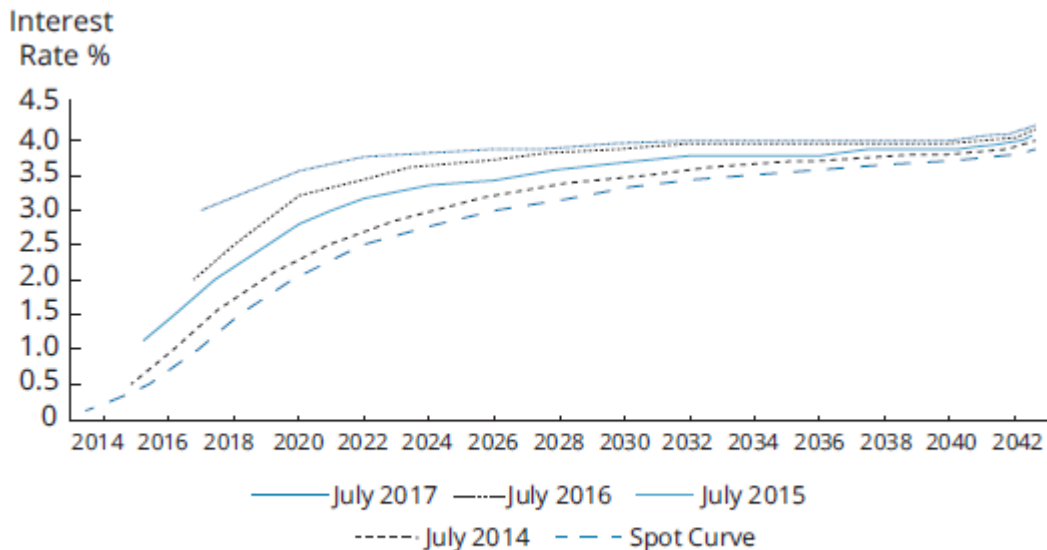
LOS 23.c: Describe the assumptions concerning the evolution of spot rates in relation to forward rates implicit in active bond portfolio management.

### Relationships Between Spot and Forward Rates

For an upward-sloping spot curve, the forward rate rises as  $j$  increases. (For a downward-sloping yield curve, the forward rate declines as  $j$  increases.) For an upward-sloping spot curve, the forward curve will be above the spot curve as shown in Figure 23.1. Conversely, when the spot curve is downward sloping, the forward curve will be below it.

Figure 23.1 shows spot and forward curves as of July 2013. Because the spot yield curve is upward sloping, the forward curves lie above the spot curve.

Figure 23.1: Spot Curve and Forward Curves



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From the forward rate model:

$$(1 + S_T)^T = (1 + S_1)[1 + f(1, T - 1)]^{(T - 1)}$$

which can be expanded to:

$$(1 + S_T)^T = (1 + S_1) [1 + f(1, 1)] [1 + f(2, 1)] [1 + f(3, 1)] \dots [1 + f(T - 1, 1)]$$

In other words, the spot rate for a long-maturity security will equal the geometric mean of the one period spot rate and a series of one-year forward rates.

### Forward Price Evolution

If the future spot rates actually evolve as forecasted by the forward curve, the forward price will remain unchanged. Therefore, a change in the forward price indicates that the future spot rate(s) did not conform to the forward curve. When spot rates turn out to be

lower (higher) than implied by the forward curve, the forward price will increase (decrease). A trader expecting lower future spot rates (than implied by the current forward rates) would purchase the forward contract to profit from its appreciation.

For a bond investor, the return on a bond over a one-year horizon is always equal to the one-year risk-free rate *if the spot rates evolve as predicted by today's forward curve*. If the spot curve one year from today is not the same as that predicted by today's forward curve, the return over the one-year period will differ, with the return depending on the bond's maturity.

An active portfolio manager will try to outperform the overall bond market by predicting how the future spot rates will differ from those predicted by the current forward curve.

### EXAMPLE: Spot rate evolution

Jane Dash, CFA, has collected benchmark spot rates as shown here:

Maturity	Spot Rate
1	3.00%
2	4.00%
3	5.00%

The expected spot rates at the end of one year are as follows:

Year	Expected Spot
1	5.01%
2	6.01%

Calculate the one-year holding period return of a:

1. 1-year zero-coupon bond.
2. 2-year zero-coupon bond.
3. 3-year zero-coupon bond.

### Answer:

First, note that the expected spot rates provided just happen to be the forward rates implied by the current spot rate curve.

Recall that:

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

Hence:

$$[1 + f(1,1)]^1 = \frac{(1 + S_2)^2}{(1 + S_1)} = \frac{(1.04)^2}{(1.03)} \rightarrow f(1,1) = 0.0501 \text{ and}$$

$$[1 + f(1,2)]^2 = \frac{(1 + S_3)^3}{(1 + S_1)} = \frac{(1.05)^3}{(1.03)} \rightarrow f(1,2) = 0.0601$$