2024 Level 2 - Seminars

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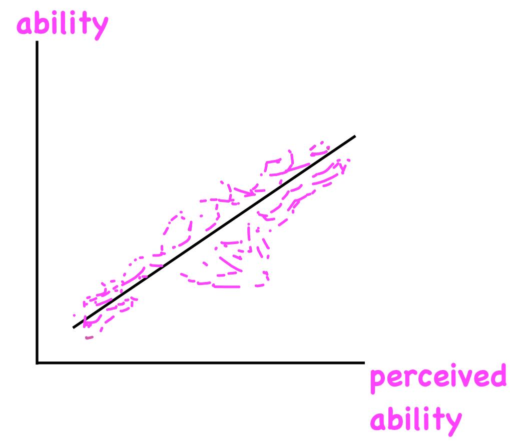
Standard Errors with CH and Multicollinearity

Conditional Heteroskedasticity

$$S_{\hat{b}} = \frac{SEE}{\sqrt{\Sigma(x_i - \bar{x})^2}} \rightarrow \sqrt{\frac{\Sigma(y_i - \bar{y})^2}{n-2}}$$

Q15-24 exh. #1 → 2.2225
exh. #2 → 2.7905 → $\frac{2.7905}{\sqrt{2.2225}} = 1.8718 = SE_{\hat{b}_1}$

(Debt ratio)



Multicollinearity

$y = b_0 + b_1X_1 + b_2X_2 + b_3X_3$

$$S_{b_1} = \sqrt{\frac{1 - R^2}{(1 - R^2_{X_kG_k})(N - k - 1)}} \cdot \frac{S_y}{S_x}$$

↓

$X_1 = b_0 + b_1X_2 + b_2X_3$

↓

calculate R^2

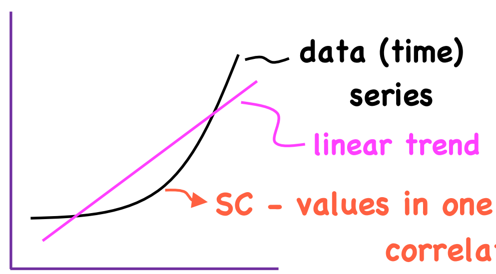
$(t) = b_1\downarrow / SE\uparrow$

higher correlation between X_1 and X_2 & X_3, the higher $R^2_{X_kG_k}$, the lower $(1 - R^2_{X_k,G_k})$, the lower the denominator, the larger the SE

(+) - the smaller b_1 will be (partial coefficient)

Time Series Issues (SC, RW, S and CH)

- Time Series is just linear regression
  - however, the assumptions of linear regression are usually not satisfied



- for non-AR(p) models, can use DW test

SC - values in one period appear to be correlated with values in another period

the DV & IV are not distinct in TS

∴ SC is serious (estimated coefficients will not be consistent)

Step #1: use AR(1) ⇒ $X_t = b_0 + b_1 X_{t-1}$ (DV: X_t; IV: X_{t-1})

$X_t = b_0 + b_1 X_{t-1} + \varepsilon$ AR(1)

But: new problem

⇒ must be Covariance Stationary
(constant & finite mean & variance)

- first differencing
$|b_1| < 1$

- for SC & AR(p) models, use autocorrelations of the residual
(standard output from stat. software)

- looking for insignificant t-stats (H₀: autocorrelations = 0)

Z → 1.96 (< ~2.0) $\alpha = .05$

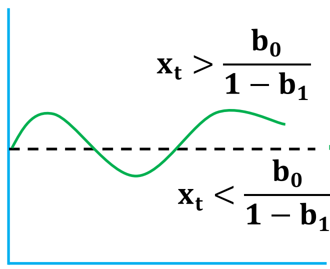
- no SC.

Lag	Autocorrelation	Standard Error	t-stat.	
1	AC_1	$1/\sqrt{T}$	$AC_1/(1/\sqrt{T})$	< crit.-t
2	AC_2	⋮	$AC_2/(1/\sqrt{T})$	< crit.-t
3	⋮	⋮	⋮	
4	⋮	⋮	$AC_4/(1/\sqrt{T})$ →	> crit.-t
⋮	⋮	⋮		
12	AC_{12}		$AC_{12}/(1/\sqrt{T})$	

Random Walks/

- first ⇒ mean reversion
  - all Cov. St. TS have a mean reverting level
- at a mean reverting level, $x_{t+1} = x_t$

$$= b_0 + b_1 x_t$$



$x_t > \frac{b_0}{1 - b_1}$ → x_{t+1} will decrease (vs. x_t)

→ mean reverting level $x_t = \frac{b_0}{1 - b_1}$

$x_t < \frac{b_0}{1 - b_1}$ x_{t+1} will increase (vs. x_t)

Random Walk → $x_{t+1} = x_t + \varepsilon_t$

(b_0 = 0, b_1 = 1)

- undefined mean reverting level
- no finite variance, ∴ not Cov. St.

ε - constant variance

$\rho_{\varepsilon_t \varepsilon_{t-1}} = 0$

Random Walks/

⇒ We cannot use standard regression analysis on a time series that is a random walk

- first differencing: new series $y_t = \Delta x_t = x_t - x_{t-1}$

RW: b_0 = 0 b_1 = 0 , $y_t = \varepsilon_t$

- mean reverting level = $\frac{0}{1 - 0}$ = 0

Cov. St. TS	Non-Cov. St. TS
- profit margins	- quarterly sales
- inflation rates	- CPI, GDP
- Dividend yields	- EPS
	- prices

must first difference

- if RW:
  - b_0 = 0, b_1 = 0 } non-sig. t-stats.
  - no SC in ε_t

if b_0 ≠ 0, b_1 = 0 } RW w/ drift.

- Example #10

Random Walks/

- Unit Root test of Nonstationarity/

Dickey-Fuller test for a unit root (DF) (H_0 = unit root)

AR(1) ⇒ $| b_1 | < 1$ = Cov. St.

(i.e. not first diff.) $| b_1 | = 1$ = not Cov. St. , RW, unit root

- issue ⇒ if $|b_1| = 1$, then x_t is not Cov. St. & t-stat will not follow a t-distribution (invalid t-tests)

Seasonality/ - so seriously super simple

- the TS shows regular patterns of movement within the year → quarterly (4^{th} Q) → monthly (12^{th} M) } identified by a sig. autocor. of the error term.
- just use a seasonal lag → X_{t-4}, X_{t-12}

Example #16

Conditional Heteroskedasticity/ ARCH

Heteroskedasticity: variance of error term depends on the value of the IV

(SE will be incorrect)

- typically underestimated

$$\varepsilon_t^2 = a_0 + a_1\varepsilon_{t-1}^2 + \mu_1$$

⇒ estimate this equation (requires the residuals from a previously estimated regression)

↓ if $a_1 \neq 0$ TS is ARCH(1)

⇒ Example 17

Currency Exchange Rate

$1.25 → 1.35 $1.99 → 2.09 $1.24 → 1.25

(direct)

$\text{CAD}/_{\text{COKE}} = 1.25$ $\text{CAD}/_{\text{BREAD}} = 1.99$ $\text{CAD}/_{\text{USD}} = 1.24$

⇒ What you get for $1

$\text{COKE}/_{\text{CAD}} = {}^{1}/_{1.25} = .80$ $\text{BREAD}/_{\text{CAD}} = {}^{1}/_{2.09} = .4785$ $\text{USD}/_{\text{CAD}} = {}^{1}/_{1.25} = .80¢\ \text{USD}$

(indirect)

Exchange Rates/

$^{\text{P}}/_{\text{B}}$ ⇒ direct $^{d}/_{f}$ e.g. $^{\text{CAD}}/_{\text{USD}} = 1.2500$ $1.25 for a D-coke

⇒ indirect $^{f}/_{d}$ $^{\text{USD}}/_{\text{CAD}} = .8000$ .8 D-Coke for $1 CAD

changes/

t_0 $^{\text{P}}/_{\text{B}} = 1.25$ $\frac{1.30}{1.25} - 1 = 4\%$ • the B ↑ 4%

t_1 $^{\text{P}}/_{\text{B}} = 1.30$ $\frac{{}^{1}/_{1.30}}{{}^{1}/_{1.25}} - 1 = \frac{.7692}{.80} - 1 = -3.85\%$ • the P ↓ 3.85%

bid-offer/

	B	O
$^{\text{CAD}}/_{\text{USD}}$	1.2498	1.2502
$^{\text{USD}}/_{\text{CAD}}$	$\frac{1}{1.2502}$ = .7999	$\frac{1}{1.2498}$ = .8001

Real exchange rate: $S_{d/f}$ CPI_f CPI_d

e.g./ $CAD/_{USD} = 1.2500$ ⇒ for \$1.25 CAD, we get \$1 USD of purchasing power

→ Price level in US ↑ 2%

1.2500 × 1.02 = 1.275 ⇒ for \$1.275 CAD, we get \$1 USD of purchasing power

- now assume the CAD price level ↑ by 1%

$\frac{1.275}{1.01} = 1.2624$ ⇒ for every \$1.2624 CAD, we get \$1 USD of purchasing power

absolute PPP

$$\frac{S_{d/f} \times P_f}{P_d} = S_{d/f} \times \left(P_f/P_d\right) \quad \text{or/} \quad \frac{S_{P/B} \times CPI_B}{CPI_P} = S_{P/B} \cdot \left(\frac{CPI_B}{CPI_P}\right)$$

cross rates $A/_B \times B/_C = A/_C$

e.g./ $CAD/_{USD} = 1.2493$ $USD/_{EUR} = 1.2220$

$$CAD/_{EUR} = CAD/_{USD} \times USD/_{EUR} = 1.2493 \times 1.2220 = 1.5266$$

- if we have $A/_B$ & $A/_C$, and we want $C/_B$, must first invert $A/_C$

e.g./ $CAD/_{USD} = 1.2493$ $CAD/_{JPY} = .01115$

$$JPY/_{USD} = CAD/_{USD} \times JPY/_{CAD}$$

$$= 1.2493 \times 1/_{.01115} = 1.2493 \times 89.686$$

$$= 112.04$$

- Decompose: $CHF/_{EUR}$

$$= USD/_{EUR} \times CHF/_{USD}$$

	bid	offer
EUR/USD	€1.000	€1.0015
GBP/USD	£2.0000	£2.0100
EUR/GBP	€ .3985	€ .4000

Arb. opportunity?

$EUR/_{USD} \times USD/_{GBP}$ vs.

$1.000 \times 1/_{2.0100} = .4975$ .3985

$1.0015 \times 1/_{2.000} = .50075$ .4000 buy low

should be is

Step 1 Sell $1000 USD for EUR
→ Dealer buys USD @ €1.000
sells USD @ €1.0015

1000 x 1.000 = €1000

Step #2 Sell €1000 EUR for GBP = buy GBP
- Dealer buys GBP for € .3985
sells GBP for € .4000

$€1000/_{€.400} = £2500$

Step #3. Sell GBP for USD (= buy USD)
Dealer buys USD for £2.000
sells USD for £2.010

$2500/_{2.010} = 1243.78$

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	bid	offer
EUR/USD	€1.000	€1.0015
GBP/USD	£2.0000	£2.0100
EUR/GBP	€ .5985	€ .6000

Arb. opportunity?

$EUR/_{USD} \times USD/_{GBP}$ vs.

$1.000 \times 1/_{2.0100} = .4975$ .5985 sell high

$1.0015 \times 1/_{2.000} = .50075$ .6000

should be is

Step 1 Sell $1000 USD for GPB
→ Dealer buys USD @ £2.000
sells USD @ £2.0100

1000 x 2.000 = £2,000

Step #2 Sell 2000 GPB for EUR
- Dealer buys GBP for € .5985
sells GBP for € .6000

2000 x .5985 = €1197

Step #3. Sell EUR for USD (= buy USD)
Dealer buys USD for €1.0000
sells USD for €1.0015

$1197/_{1.0015} = 1195.21$

forward rates/ $r_{CAD} = 1\%$ $r_{USD} = 3\%$

$CAD/_{USD} = 1.2500$ $USD/_{CAD} = .8000$

$\frac{1.2500(1.01)}{1.03} = 1.2257$ $\frac{.8000(1.03)}{1.01} = .8158$

$1/_{1.2257} = .8158$

$F_{d/f} = S_{d/f}\left(\frac{1+r_d}{1+r_f}\right)$

- I will have USD in the future, locking in a CAD rate

$F_{f/d} = S_{f/d}\left(\frac{1+r_f}{1+r_d}\right)$

- I will have CAD in the future, locking in a US rate

⇒ quoted: Spot + points → forward premium $(1+r_P) > (1+r_B)$

Spot - points → forward discount $(1+r_P) < (1+r_B)$

- Bought $1M CAD for delivery against the USD in 1 yr.

? ⇒ $CAD/_{USD} = .8000/.8004$ $USD/_{CAD} = 1.2493/1.2500$

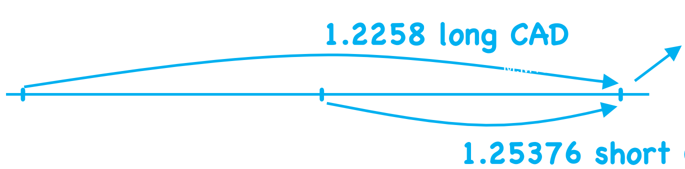
1 yr. ⇒ Forward points 158.4/162.5 -242.7/-242.8

F_0 = ?

6-mos. later $S_{USD/CAD} = 1.2631/1.2634$ $r_{CAD} = 2.75\%$ $r_{USD} = 1.25\%$

6 mos. ⇒ f-points -93.4/-93.5

? Sell / Buy $1M CAD / USD



(1.25376 - 1.2258)$1M = 27,960 USD / CAD ?

CIRP
|
hedged

$$F_{f/d} = S_{f/d}\left(\frac{1 + r_f}{1 + r_d}\right)$$

UIRP
|
unhedged

$$\%\Delta S^e_{f/d} \cong (i_f - i_d)$$

Forward Rate Parity

- the forward rate will be an unbiased predictor of the future spot fx.-rate

i.e. $F_{f/d} = S^e_{f/d}$

Absolute PPP

$$S_{f/d} = \frac{CPI_f}{CPI_d}$$

Relative PPP

$$\%\Delta S_{f/d} \cong (\pi_f - \pi_d)$$

ex-ante PPP

$$\%\Delta S^e_{f/d} \cong (\pi^e_f - \pi^e_d)$$

$$i_f - i_d = (r_f - r_d) + (\pi^e_f - \pi^e_d)$$

$$(r_f - r_d) = (i_f - i_d) - (\pi^e_f - \pi^e_d)$$

$(r_f - r_d) \downarrow$ 0; $(i_f - i_d) \downarrow$ $\%\Delta S^e_{f/d}$; $(\pi^e_f - \pi^e_d) \downarrow$ $\%\Delta S^e_{f/d}$

$$i_f - i_d = \pi^e_f - \pi^e_d$$

if UIRP + ex-ante hold, then real int. rate parity holds