LM01 Rates and Returns

Ver 1.0

Introductory Note

Financial Calculator: CFA Institute allows only two calculator models during the exam:

- Texas Instruments BA II Plus (including BA II Plus Professional) and
- Hewlett Packard 12C (including the HP 12C Platinum, 12C Platinum 25th anniversary edition, 12C 30th anniversary edition, and HP 12C Prestige)

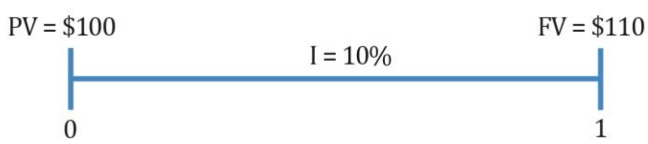
Unless you are already comfortable with the HP financial calculator, we recommend using the Texas Instruments financial calculator. Explanations and keystrokes in our study materials are based on the Texas Instruments BA II Plus calculator.
Before you start using the calculator to solve problems, we recommend that you set the number of decimal places to 'floating decimal'.

1. Introduction

If you have $100 today, versus an option to receive $100 after three years, what would you prefer?

Obviously, you would prefer $100 today. Even though you have the same amount ($100) in both cases, you prefer $100 today. This means that there has to be some value associated with time, because you are putting more value on the $100 that you are getting today, relative to the $100 at a later point in time. This is known as 'time value of money.'

Let us say that you are indifferent between $100 dollars today versus $ 110 after one year.



Present value (PV): The money today or the value today is called the present value (PV = 100). This could be an investment which you make at time 0.

Future value (FV): The value at a future point in time is called the future value (FV = 110).

Interest rate (I): The relationship or the link between present value and future value is established through an interest rate (I = 10%).

In this reading we will cover the meaning and interpretation of interest rates, and learn how to calculate, interpret, and compare different return measures.

2. Interest Rates and Time Value of Money

Let's discuss the different interpretations of interest rates using an example. Say you lend \$900 today and receive \$990 after one year (–ve sign indicates outflow).

Interest rates can be interpreted as:

1. Required rate of return: The fact that you are willing to give \$900 today on the condition that you get \$990 after one year means that to engage in this transaction, you require a return of 10%. (Simple calculation will show you that the interest rate in this transaction is 10%).
2. Discount rate: You can discount the money that you will receive after one year i.e. \$990 at 10% to get the present value of \$900 (990/1.1 = 900). Therefore, the 10% can also be thought of as a discount rate.
3. Opportunity cost: Let's say instead of lending the \$900, you spent it on something else. You have then forgone the opportunity to earn 10% interest. Therefore, 10% can also be thought of as an opportunity cost.

Determinants of Interest Rates

As an investor, we can think of the interest rate as a sum of the following components:

Interest rate = Real risk-free interest rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium

Let's look at the different components.

- Real risk-free interest rate: This is the rate that you get on a security that has no risk and is extremely liquid. We make an assumption here that there is no inflation.
- Inflation premium: We can then add on an inflation premium. Inflation premium is the expected annual inflation in the upcoming period.
- Default risk premium: We can also then add a default risk premium. This is the additional premium that investors require because of the risk of default.
  Example: Let's say that you lend \$100 each to person A and person B. However, B has a high risk of default, so you are worried that he might not pay. Therefore you might demand a higher return from B as compared to A, because of the risk of default. This additional return that you demand is called the default risk premium.
- Liquidity premium: Liquidity premium compensates investors for the risk of receiving less than the fair value for an investment if it must be converted to cash quickly.

Example: Think of two investments C and D which are similar in all regards. The only difference is that investment C is extremely liquid, whereas investment D is not that liquid. Clearly as investors, we will demand a higher return on D because it is not easy to sell. This additional return that we demand is called the liquidity premium.

- Maturity premium: Finally, we have the maturity premium. This is the premium that investors demand on a security with long maturity. The maturity premium compensates investors for the increased sensitivity of the market value of debt to a change in market interest rates as maturity is extended.
  Example: Let's say we have two securities, E and F. Security E has a maturity of 1 year and security F has a maturity of 4 years. Because of the longer maturity, F has more risk, in terms of its price being more sensitive to changes in interest rate.
  Instructor's Note: You will understand this concept better when you study fixed income securities. But for now, you can take it as a given that F has higher risk because of the longer maturity.
  Obviously, investors will demand some compensation for the higher level of risk. This additional return that investors demand is called the maturity premium.

Nominal risk-free rate:
Nominal risk-free rate = Real risk-free interest rate + Inflation premium.
So if the real risk-free rate is 3% and the inflation premium is 2%, then the nominal risk-free rate is 5%.

Instructor's Note: On the exam if you get a term 'risk-free rate' with no mention of whether the rate is real or nominal, then the assumption is that we are talking about the nominal risk-free rate.

Example

Investments	*Maturity (in years)*	*Liquidity*	*Default risk*	*Interest Rates(%)*
A	1	High	Low	2.0
B	1	Low	Low	2.5
C	2	Low	Low	r
D	3	High	Low	3.0
E	3	Low	High	4.0

1. Explain the difference between the interest rates on Investment A and Investment B.
2. Estimate the default risk premium.
3. Calculate upper and lower limits for the interest rate on Investment C, r.

Solution:

1. Investments A and B have the same maturity and the same default risk. However, B has a lower liquidity as compared to A. Hence, investors will demand a liquidity premium on B.

The difference between their interest rates i.e. 2.5 – 2.0 = 0.5% is equal to the liquidity premium.

2. Consider investments D and E, they have the same maturity, but different liquidity and different default risk. Let's make liquidity the same and create a new low liquidity version of D. This version will have a higher interest rate, because now investors will demand a liquidity premium. We have already determined that the liquidity premium is 0.5%. Therefore, the low liquidity version of D will have an interest rate of 3.0 + 0.5 = 3.5%.
   Now compare this version of D with investment E. The only difference between the two is default risk. E has a higher default risk. Therefore, the difference between their interest rates i.e. 4.0 – 3.5 = 0.5% must be equal to the default risk premium.
3. Notice that between B and C, the only difference is that C has a longer maturity. Therefore, interest rate of C must be higher than B (2.5%).
   Also notice that between C and the low liquidity version of D, the only difference is that C has a shorter maturity. Therefore, interest rate on C has to be lower than the low liquidity version of D (3.5%).
   So, the range for C is $2.5 < r < 3.5$.

3. Rates of Return

A financial asset's total return consists of two components: income and capital appreciation.

Holding Period Return

Holding period return (HPR) is the return that an investor earns over a specified holding period. The holding period can range from days to years. The formula for calculating HPR for an investment that makes one-time cash payment at the end of the holding period is given below:

$$HPR = \frac{P_1 - P_0 + D_1}{P_0} = \frac{\text{ending value} - \text{beginning value} + \text{cash flow}}{\text{beginning value}}$$

where:
P_0 = initial investment
P_1 = price received at the end of the holding period
D_1 = cash paid by the investment at the end of the holding period

Example

Assume we buy a stock for \$50. Six months later, the stock price goes up to \$53 and we receive a dividend of \$2. Calculate the holding period return.
Solution:
The return for the six-month holding period is given below:

$$HPR = \frac{53 + 2 - 50}{50} = 0.10 = 10\%$$

Sometimes a holding period return can be computed for a period longer than a year. For example, an analyst may need to calculate the holding period return from three annual returns. In this case, the holding period return is calculated as:

$R = [(1 + R_1) \times (1 + R_2) \times (1 + R_3)] – 1$

where:
R_1, R_2, and R_3 are the three annual returns

Example:

The annual returns of a mutual fund for the past three years are presented below.

2020	20%
2021	-8%
2022	-1%

Calculate the fund's holding period return over the three-year period.

Solution:

$R = [(1 + R_1) \times (1 + R_2) \times (1 + R_3)] – 1$

$R = [(1 + 0.20) (1 – 0.08) (1 – 0.01)] – 1 = 0.0929 = 9.296\%$

Arithmetic or Mean Return

The arithmetic mean is the sum of the observations divided by the number of observations. It is expressed as:

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

For example, if the annual returns of a mutual fund for the past three years are 20%, - 8%, and -1%, the mean return is $\frac{20-8-1}{3} = 3.66\%$

A drawback of the arithmetic mean is that it is sensitive to extreme values (outliers). It can be pulled sharply upward or downward by extremely large or small observations, respectively.

<u>Outliers</u>

When data contains outliers, there are three options to deal with the extreme values:

Option 1: Do nothing; use the data without any adjustment.

Option 2: Delete all the outliers.

Option 3: Replace the outliers with another value.

Option 1 is appropriate in cases when the extreme values are genuine.

Option 2 excludes extreme observations. A *trimmed mean* excludes a stated percentage of the lowest and highest values and then calculates the arithmetic mean of the remaining values.

Option 3 replaces extreme observations with observations closest to them. A *winsorized mean* assigns a stated percentage of the lowest values equal to one specified low value and a stated percentage of the highest values equal to one specified high value, and then computes a mean from the restated data.

Geometric Mean Return

The geometric mean is calculated as the n^{th} root of a product of n numbers. The most common application of the geometric mean is to calculate the average return of an investment. The formula is:

$$R_G = [(1 + R_1)(1 + R_2) \ldots (1 + R_n)]^{\frac{1}{n}} - 1$$

Example

The return over the last four periods for a given stock is: 10%, 8%, -5% and 2%. Calculate the geometric mean.

Solution:

$$[(1 + 0.10)(1 + 0.08)(1 - 0.05)(1 + 0.02)]^{\frac{1}{4}} - 1 = 0.0358 = 3.58\%$$

Given the returns shown above, $1 invested at the start of period 1 grew to:

$1.00 x 1.10 x 1.08 x 0.95 x 1.02 = $1.151. If the investment had grown at 3.58% every period, $1.00 invested at the start of period 1 would have increased to:

$1.00 x 1.0358 x 1.0358 x 1.0358 x 1.0358 = $1.151. As expected, both scenarios give the same answer. 3.58% is simply the average growth rate per period.

Other applications of the geometric mean involve the use of a second formula:

$$\ln\overline{X}_G = \frac{\sum_{i=1}^{n} \ln X_i}{n}$$

Instructor's Note: This formula is less testable.

Example:

The P/E ratio of a stock over the past four years has been: 10, 15, 14, 13. Calculate the geometric mean P/E.

Solution:

$$\ln\overline{X}_G = \frac{\sum_{i=1}^{n} \ln X_i}{n}$$

$$\ln\overline{X}_G = \frac{\ln 10 + \ln 15 + \ln 14 + \ln 13}{4} = 2.55$$

$\overline{X}_G = e^{2.55} = 12.807$

Using Geometric and Arithmetic Means

The geometric mean is appropriate to measure past performance over multiple periods.

Example
The portfolio returns for the past two years were 100% in year 1 and -50% in year 2. What was the mean return?

Solution:
Past return = geometric mean = $((1+1.0) \times (1-0.5))^{0.5} - 1 = 0\%$

The arithmetic mean is appropriate for forecasting single period returns.

Example
Two possible returns for the next year are 100% and -50%. What is the expected return?

Solution:
Expected return = Arithmetic mean = (100 – 50)/2 = 25%

The Harmonic Mean

The harmonic mean is a special type of weighted mean in which an observation's weight is inversely proportional to its magnitude. The formula for a harmonic mean is:

$$X_H = \frac{n}{\sum_{i=1}^{n} \frac{1}{X_i}}$$

where: $X_i > 0$ for i = 1, 2 … n, and n is the number of observations

The harmonic mean is used to find average purchase price for equal periodic investments.

Example
An investor purchases $1,000 of a security each month for three months. The share prices are $10, $15 and $20 at the three purchase dates. Calculate the average purchase price per share for the security purchased.

Solution:

The average purchase price is simply the harmonic mean of $10, $15 and $20.

The harmonic mean is:

$$\frac{3}{\frac{1}{\$10} + \frac{1}{\$15} + \frac{1}{\$20}} = \$13.85.$$

A more intuitive way of solving this is total money spent purchasing the shares divided by the total number of shares purchased.

Total money spent purchasing the shares = $1,000 x 3 = $3,000

Total shares purchased = sum of shares bought each month

$= \frac{\$1,000}{10} + \frac{\$1,000}{15} + \frac{\$1,000}{20}$

= 100 + 66.67 + 50 = 216.67

Average purchase price per share $= \frac{\$3,000}{216.67} = \$13.85$

Comparison of AM, GM and HM

- Arithmetic mean × Harmonic mean = Geometric mean^2
- If the returns are constant over time: AM = GM = HM.
- If the returns are variable: AM > GM > HM.
- The greater the variability of returns over time, the more the arithmetic mean will exceed the geometric mean.

Which mean to use?

- Arithmetic mean: Should be used with single period or cross-sectional data.
- Geometric mean: Should be used with time-series data.
- Harmonic mean: Should be used to find average purchase price for equal periodic investments.
- Trimmed mean: Should be used when the data has extreme outliers.
- Winsorized mean: Should be used when the data has extreme outliers.

4. Money-Weighted and Time-Weighted Return

Internal Rate of Return

The internal rate of return (IRR) is the discount rate that makes the net present value of an investment equal to zero. It is 'internal' because it depends only on the cash flows of the investment; no external data is needed. The formula for IRR is as follows:

$$NPV = CF_0 + \left[\frac{CF_1}{(1 + IRR)^1}\right] + \left[\frac{CF_2}{(1 + IRR)^2}\right] + \dots + \left[\frac{CF_N}{(1 + IRR)^N}\right] = 0$$

where:
CF_0 = usually the initial investment which is a cash outflow
CF_t = the expected net cash flow at time t
NPV = net present value of the investment
IRR = internal rate of return

The IRR is a single number which represents the return generated by an investment. Consider a very simple example where the initial investment is $100. One year later the amount received from this investment is $110. There are no other cash flows. If we apply the formula for IRR we get:

$$-100 + \frac{110}{1 + \text{IRR}} = 0$$

Solving this equation shows that IRR = 0.1 or 10%.

Example

Consider an initial investment of $150,000. Estimated cash flows for the following three years from this investment are $50,000, $100,000 and $40,000 respectively. What is the IRR?

Solution:

We can set up an equation with the initial outflow equal to the present value of future cash flows and solve for the IRR:

$$\text{CF0} = \left[\frac{\text{CF1}}{(1 + \text{IRR})^1}\right] + \left[\frac{\text{CF2}}{(1 + \text{IRR})^2}\right] + \left[\frac{\text{CF3}}{(1 + \text{IRR})^3}\right]$$

Plugging in the values, we get:

$$\$150{,}000 = \left[\frac{\$50{,}000}{(1 + \text{IRR})^1}\right] + \left[\frac{\$100{,}000}{(1 + \text{IRR})^2}\right] + \left[\frac{\$40{,}000}{(1 + \text{IRR})^3}\right]$$

While it is theoretically possible to solve the above equation, it is much simpler to use the financial calculator.

Keystrokes	*Explanation*	*Display*
[2nd] [QUIT]	Return to standard mode	0
[CF] [2nd] [CLR WRK]	Clear CF Register	CF = 0
150 [+/-] [ENTER]	Initial outlay (in 000's)	CF0 = -150
[↓] 50 [ENTER]	Enter CF at t = 1	C01 = 50
[↓] [↓] 100 [ENTER]	Enter CF at t = 2	C02 = 100
[↓] [↓] 40 [ENTER]	Enter CF at t = 3	C03 = 40
[↓] [ÌRR] [CPT]	Compute IRR	13.11%

Money-Weighted Rate of Return

The money-weighted rate of return is the internal rate of return (IRR) of an investment.

Let us consider a simple example to illustrate this point. At time t = 0, an investor buys a share for $20.00. At the end of the Year 1, he receives a dividend of $0.50 and purchases another share for $22.50. At the end of the Year 2, he sells both shares for $23.50 each after receiving another dividend of $0.50 per share. What is the money-weighted return?

Since the money-weighted return is the IRR, we can use a financial calculator. The first step is to determine the net cash flows for every period. This is illustrated in the table below:

Time (end of period)	*Outflow (-)*	*Inflow (+)*	*Net cash flow*
0	-$20.00 To purchase the first share		-$20.00

1	-$22.50 To purchase the second share	$0.50 Dividend received on first share	-$22.00
2		Dividend received = $0.50 x 2 shares = $1.00 From sales of 2 shares = $47	+48.00

After entering these cash flows(CF0 = -20, CF1 = -22, and CF3 = 48), use the calculator's IRR function to find the money-weighted rate of return as 9.39%.

Time-Weighted Rate of Return

The time-weighted rate of return measures the compound growth rate of $1 initially invested in the portfolio over a stated measurement period. The time-weighted return can be calculated using the following steps:

1. Break the overall evaluation period into sub-periods based on the dates of cash inflows and outflows.
2. Calculate the holding period return on the portfolio for each sub-period.
3. Link or compound holding period returns to obtain an annual rate of return for the year (the time-weighted rate of return for the year).
4. If the investment is for more than a year, take the geometric mean of the annual returns to obtain the time-weighted rate of return over that measurement period.

Consider the same example we discussed above with the following cash flows:

Time	*Outflow*	*Inflow*
0	$20.00 to purchase the first share	
1	$22.50 to purchase the second share	$0.50 dividend received on first share
2		$1.00 dividends ($0.50 x 2 shares); $47.00 from selling 2 shares @ $23.50 per share

Calculating the TWRR for this example is relatively simple because cash flows only occur at the start/end of every year. We will follow the steps mentioned earlier:

<u>Steps 1</u>: Break into evaluation period and value the portfolio at start/end of every period.

- Value of the portfolio at the start of Year 1 (t = 0) is $20.00.
- Value of portfolio at the end of Year 1 (t = 1) before the purchase of the new share is 22.50 + 0.50 = $23.00. Note that the dividend of $0.50 on the first share is received at the end of Year 1.
- Value of the portfolio at the start of Year 2 (t = 1) after the purchase of the second share is 22.50 + 22.50 = $45.00. The dividend of $0.50 from the first share is paid out and is not considered as part of the portfolio.
- Value of the portfolio at the end of Year 2 (t = 2) is 23.50 + 23.50 + 0.50 + 0.50 = $48.00. Both shares pay a dividend of $0.50 at the end of the second year.

Step 2: Calculate the holding period return on the portfolio for each sub-period.

- In this question the cash flows are taking place at the start/end of each period. Hence there are no sub-periods. *Scenarios involving sub-periods will be covered in the next example.*

Step 3: Link or compound holding period returns to obtain an annual rate of return for the year.

- The annual rate of return is based on the portfolio value at the start and end of each period.
- The portfolio value at the start of Year 1 was $20.00 and the value at the end of Year 1 was $23.00. Hence the holding period return was 15.00%.
- The portfolio value at the start of Year 2 was $45.00 and the value at the end of Year 2 was $48.00. Hence the holding period return was 6.67%.

Step 4: If the investment is for more than a year, take the geometric mean of the annual returns to obtain the time-weighted rate of return over that measurement period.

The TWRR is calculated as: $(1.15 * 1.067)^{\frac{1}{2}} - 1 = 0.1077 = 10.77\%$.

Example

Consider an investment where cash flows occur at the start/end of every quarter. Here each quarter is considered a sub-period. The return for each sub-period has already been calculated and is shown below:

Calculate the time-weighted return.

Solution:

Step 1 (break evaluation into sub-periods) and step 2 (calculate HPR for sub-periods) have been done for you.

Step 3: Link the quarterly returns to determine the return for Years 1 and 2 respectively.
For year 1: (1 + 0.10) (1 – 0.05) (1 + 0.15) (1 – 0.10) = 1.0816
For year 2: (1 – 0.20) (1 + 0.30) (1 + 0.20) (1 + 0) = 1.2480

Step 4: Determine the annualized return by taking the geometric mean.
TWRR = $(1.0816 \times 1.2480)^{1/2} - 1 = 0.1618 = 16.18\%$.

Money-weighted v/s time-weighted returns

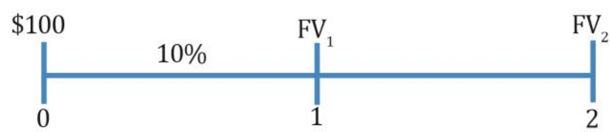
- The money-weighted rate of return is impacted by the timing and amount of cash flows.

- The time-weighted rate of return is not impacted by the timing and amount of cash flows.
- The time-weighted return is an appropriate performance measure if the portfolio manager does not control the timing and amount of investment.
- On the other hand, money-weighted return is an appropriate measure if the portfolio manager has control over the timing and amount of investment.

5. Annualized Return

Annual compounding

Let's understand this concept with a simple example.
Say present value (PV) = $100 and interest rate (r) = 10%.
What is the future value (FV) after one year?
What is the future value (FV) after two years?



The future value of a single cash flow with annual compounding can be computed using the following formula:

$$FV_N = PV(1 + r)^N$$

where:
FV_N = future value of the investment
N = number of periods
PV = present value of the investment
r = rate of interest

Therefore,

$$FV_1 = 100(1 + 0.1)^1 = \$110$$

$$FV_2 = 100(1 + 0.1)^2 = \$121$$

Notice that with compound interest, after two years we have $121. Whereas, with simple interest, after two years we would have $120. The difference between the two values ($1) represents the interest on interest component. In Year 2, we not only receive interest on the $100 principal, but we also receive interest on the $10 interest earned in Year 1 that has been reinvested.

Example

Cyndia Rojers deposits $5 million in her savings account. The account holders are entitled to a 5% interest. If Cyndia withdraws cash after 2.5 years, how much cash would she *most likely* be able to withdraw?

Solution:

$FV_N = PV(1 + r)^N$

$FV_{2.5} = 5(1 + 0.05)^{2.5}$ = $5.649 million

FV Calculation using a Financial Calculator

You will often use the following keys on your TI BA II Plus calculator:
N = number of periods
I/Y = rate per period
PV = present value
FV = future value
PMT = payment
CPT = compute

One important point to note is the signs used for PV and FV. If the value for PV is negative "-", then the value for FV is positive "+". An inflow is often represented as a positive number, while outflows are denoted by negative numbers.
Before you begin, set the number of decimal points on your calculator to 9 to increase accuracy.

Keystrokes	**Explanation**	**Display**
[2nd] [FORMAT] [ENTER]	Get into format mode	DEC = 9
[2nd] [QUIT]	Return to standard calc mode	0

Question: You invest $100 today at 10% compounded annually. How much will you have in 5 years?

The key strokes to compute the future value of a single cash flow are illustrated below.

Keystrokes	**Explanation**	**Display**
[2nd] [QUIT]	Return to standard calc mode	0
[2nd] [CLR TVM]	Clears TVM Worksheet	0
5 [N]	Five years/periods	N = 5
10 [I/Y]	Set interest rate	I/Y = 10
100 [PV]	Set present value	PV = 100
0 [PMT]	Set payment	PMT = 0
[CPT] [FV]	Compute future value	FV = -161.05

Non-annual Compounding

When our compounding frequency is not annual, we use the following formula to compute future value:

$$FV_N = PV\left(1+\frac{r_s}{m}\right)^{mN}$$

where:
r_s = the stated annual interest rate in decimal format
m = the number of compounding periods per year
N = the number of years

Let's understand this concept using an example.

You invest $80,000 in a 3-year certificate of deposit. This CD offers a stated annual interest rate of 10% compounded quarterly. How much will you have at the end of three years?

Solution:
There are two methods to solve this question.
<u>Formula Method</u>
PV is $80,000.
The stated annual rate is 10%.
The number of compounding periods per year is 4. The total number of periods is 4 x 3 = 12.
Therefore, future value after 12 quarters (3 years) is

$$FV_{12} = \$80{,}000\left(1+\frac{0.1}{4}\right)^{4\times3} = \$107{,}591$$

<u>Calculator Method</u>
You can also solve this problem using a financial calculator; the key strokes are given below:
N = 12, I/Y = 2.5%, PV = $80,000, PMT = 0, CPT FV = -$107,591
PMT is 0 because there are no intermediate payments in this example.

Example

Donald invested $3 million in an American bank that promises to pay 4% compounded daily. Which of the following is *closest* to the amount Donald receives at the end of the first year? Assume 365 days in a year.

A. $3.003 million
B. $3.122 million
C. $3.562 million

Solution
The correct answer is B.
<u>Formula Method</u>

$$FV_N = PV\left(1+\frac{r_s}{m}\right)^{mN}$$

$$FV_{1\ year} = 3\ million\left(1 + \frac{0.04}{365}\right)^{365} = \$3.122\ million$$

Calculator Method

N = 365, I/Y = 4/365%, PV = $3 million, PMT = 0; CPT FV = -$3.122 million

Annualizing Returns

Annualized return converts the returns for periods that are shorter or longer than a year, to an annualized number for easy comparison.

$$\text{Annualized return } = \left(1 + r_{period}\right)^{c} - 1$$

Where c = number of periods in year

Example:

(*This is based on Example 13 from the curriculum.*)

An investor is evaluating the returns of three ETFs.

ETF	Time Since Inception	Return Since Inception (%)
1	146 days	4.61
2	5 weeks	1.10
3	15 months	14.35

Which ETF has the highest annualized rate of return?

Solution:

ETF 1 annualized return = $(1.0461^{365/146}) - 1 = 11.93\%$

ETF 2 annualized return = $(1.0110^{52/5}) - 1 = 12.05\%$

ETF 3 annualized return = $(1.1435^{12/15}) - 1 = 11.32\%$

ETF 2 has the highest annualized rate of return.

Continuously Compounded Returns

We covered examples with annual compounding, quarterly compounding and daily compounding. If we keep increasing the number of compounding periods until we have infinite number of compounding periods per year, then we can say that we have continuous compounding.

The continuously compounded return associated with a holding period return can be calculated as:

- Natural logarithm of one plus that holding period return, or
- Natural logarithm of the ending price over the beginning price

i.e. $R_c = \ln(1 + R_t) = \ln(P_t/P_0)$

Example:

1. If the one-week holding period return is 4%, then the equivalent continuously

compounded return is: ln (1 + 0.04) = 3.922%

2. A stock was purchased at t = 0 for $30. One period later at t = 1, the stock has a value of $34.50. The continuously compounded return over the period can be calculated as: ln (34.50/30) = 13.976%

6. Other Major Return Measures and Their Applications

Gross Return

Gross return is the return earned by an asset manager prior to deducting management fees and taxes. It measures the investment skill of a manager.

Net Return

Net return is the return earned by the investor on an investment after all managerial and administrative expenses have been accounted for. This is the measure of return that should matter to an investor.

Assume an investment manager generates $120 for every $100, and charges a 2% fee for management and administrative expenses. The gross return, in this case, is 20% and the net return is 18%.

Pre-tax and After-tax Nominal Return

The returns we saw till now were pre-tax nominal returns, i.e., before deducting any taxes or any adjustments for inflation. This is the default, unless otherwise stated.

After-tax nominal return is the return after accounting for taxes. The actual return an investor earns should consider the tax implications as well.

In the example that we saw above for gross and net return, 18% was the pre-tax nominal return. If the tax rate for the investor is 33.33%, then the after-tax nominal return will be 18(1 - 0.3333) = 12.0006%.

Real Return

Real return is the return after deducting taxes and inflation.

$(1 + r) = (1 + r_{real})(1 + \pi)$

where:

r_{real} = real rate

π = rate of inflation

r = nominal rate

In the previous example, the after-tax nominal return was 12%. Assume the inflation rate for the period is 10%. What is the real rate of return?

Using the above formula, (1 + 0.12) = (1 + r) (1 + 0.1). Solving for r, we get 1.818%.

Instructor's tip: If the answer choices are close to each other, use this formula to determine the correct answer. Else, you may use an approximation to solve for r quickly as nominal rate = real rate + inflation.

Leveraged Return

In cases, where an investor borrows money to invest in assets like bonds or real estate, the leveraged return is the return earned by the investor on his money after accounting for interest paid on borrowed money.

Summary

LO: Interpret interest rates as required rates of return, discount rates, or opportunity costs and explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk.

An interest rate is the required rate of return. If you invest $100 today on the condition that you get $110 after one year, the required rate of return is 10%.
If the future value (FV) at the end of Year 1 is $110, you can discount at 10% to get the present value (PV) of $100. Hence, 10% can also be thought of as a discount rate.
Finally, if you spent $100 on taking your spouse out for dinner you gave up the opportunity to earn 10%. Thus, 10% can also be interpreted as an opportunity cost.

Interest rate = Real risk-free interest rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium.

Nominal risk free rate= real risk free rate + inflation premium

LO: Calculate and interpret different approaches to return measurement over time and describe their appropriate uses

Holding period return is the return earned on an asset during the period it was held.

$$\text{HPR single period} = \frac{P_T - P_0 + D_T}{P_0}$$

Arithmetic return is a simple arithmetic average of returns.

Geometric mean return is the compounded rate of return earned on an investment.

$$GM = [(1 + R_1) * (1 + R_2) * \ldots\ldots * (1 + R_T)]^{\frac{1}{T}} - 1$$

The harmonic mean is a special type of weighted mean in which an observation's weight is inversely proportional to its magnitude. The harmonic mean is used to find average purchase price for equal periodic investments.

LO: Compare the money-weighted and time-weighted rates of return and evaluate the performance of portfolios based on these measures.

The money-weighted rate of return accounts for the timing and amount of all cash flows into and out of a portfolio. It is simply the internal rate of return.

The time-weighted rate of return measures the compound rate of growth of $1 initially invested in the portfolio over a stated measurement period.

Money-weighted v/s time-weighted returns

- The money-weighted rate of return is impacted by the timing and amount of cash flows.
- The time-weighted rate of return is not impacted by the timing and amount of cash flows.

- The time-weighted return is an appropriate performance measure if the portfolio manager does not control the timing and amount of investment.
- On the other hand, money-weighted return is an appropriate measure if the portfolio manager has control over the timing and amount of investment.

LO: Calculate and interpret annualized return measures and continuously compounded returns, and describe their appropriate uses.

Annualized return converts the returns for periods that are shorter or longer than a year, to an annualized number for easy comparison.

The continuously compounded return associated with a holding period return can be calculated as:

- Natural logarithm of one plus that holding period return, or
- Natural logarithm of the ending price over the beginning price

LO: Calculate and interpret major return measures and describe their appropriate uses.

Gross return is the return earned by an asset manager prior to deducting management fees and taxes. It measures investment skill.

Net return accounts for all managerial and administrative expenses is what the investor is concerned with.

Pre-tax nominal return is the return before accounting for inflation and taxes; this is the default, unless otherwise stated.

After-tax nominal return is the return after accounting for taxes.

Real return is the return after accounting for taxes and inflation.

Leveraged return is the return earned by the investor on his money after accounting for interest paid on borrowed money

